

Observational Constraints of Modified Chaplygin Gas in RS II Brane

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April 26, 2013

Received _____; accepted _____

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ABSTRACT

FRW universe in RS II braneworld model filled with a combination of dark matter and dark energy in the form of modified Chaplygin gas (MCG) is considered. It is known that the equation of state (EoS) for MCG is a three-variable equation determined by A , α and B . The permitted values of these parameters are determined by the recent astrophysical and cosmological observational data. Here we present the Hubble parameter in terms of the observable parameters Ω_{m0} , Ω_{x0} , H_0 , redshift z and other parameters like A , B , C and α . From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the χ^2 test. The best-fit values of the parameters are obtained by 66%, 90% and 99% confidence levels. Next due to joint analysis with BAO and CMB observations, we have also obtained the bounds of the parameters (B, C) by fixing some other parameters α and A . The best fit value of distance modulus $\mu(z)$ is obtained for the MCG model in RS II brane, and it is concluded that our model is perfectly consistent with the union2 sample data.

Subject headings: RS II Braneworld Model; Modified Chaplygin Gas; Observational Data; Observational Constraints.

1. Introduction

Recent cosmological observations of the SNeIa (Perlmutter et al. (1998, 1999); Riess et al. (1998, 2004)) large scale redshift surveys (Bachall et al. (1999); Tedmark et al. (2004)), the measurements of the cosmic microwave background (CMB) (Miller et al. (1999); Bennet et al. (2000)) and WMAP (Bridle et al. (2003); Spergel et al. (2003)) indicate that our universe is presently undergoing an accelerated expansion. The observational facts are not clearly described by the standard big bang cosmology with perfect fluid. The first suitable candidate which could drive the acceleration in Einstein's gravity, was the cosmological constant Λ (which has the equation of state $w_\Lambda = -1$), but till now there is no proof of the origin of Λ . In the framework of general relativity, different interesting mechanisms such as loop quantum cosmology Asthekar et al (2011), modified gravity Cognola et al (2009), higher dimensional phenomena Chakraborty et al (2010); Ranjit et al (2012), Brans-Dicke theory Brans et al (1961), brane-world model Gergely et al (2002) and so on, suggested that some unknown matters are responsible for accelerating scenario of the universe and which violates the strong energy conditions, i.e. $\rho + 3p < 0$ and which has positive energy density and sufficient negative pressure, known as dark energy Padmanabhan (2003); Sahni et al. (2000). Dark energy associated with a scalar field is called quintessence Peebles et al. (1988). It is one of the most favored candidate for producing sufficient negative pressure to drive the cosmic acceleration, in which the scalar potential of the field dominates over the kinetic term. In the present cosmic concordance Λ CDM model the Universe is formed of $\sim 26\%$ matter (baryonic + dark matter) and $\sim 74\%$ of a smooth vacuum energy component. However there is about 0.01% of thermal CMB component, but in spite of this, its angular power spectrum of temperature anisotropies encode important information about the structure formation process and other cosmic observables.

If we assume a flat universe and further assume that the only energy densities present are those corresponding to the non-relativistic dust-like matter and dark energy, then we need to know Ω_m of the dust-like matter and $H(z)$ to a very high accuracy in order to get a handle on Ω_X or w_X of the dark energy Choudhury et al. (2007); Padmanabhan et al. (2003). This can

be a fairly strong degeneracy for determining $w_X(z)$ from observations. TONRY data set with the 230 data points Tonry et al. (2003) alongwith the 23 points from Barris et al Barris et al. (2004) are valid for $z > 0.01$. Another data set consists of all the 156 points in the “gold” sample of Riess et al Riess et al. (2004), which includes the latest points observed by HST and this covers the redshift range $1 < z < 1.6$. In Einstein’s gravity and in the flat model of the FRW universe, one finds $\Omega_\Lambda + \Omega_m = 1$, which are currently favoured strongly by CMBR data (for recent WMAP results, see Spergel et al. (2003)). In a simple analysis for the most recent RIESS data set gives a best-fit value of Ω_m to be 0.31 ± 0.04 . This matches with the value $\Omega_m = 0.29^{+0.05}_{-0.03}$ obtained by Riess et al Riess et al. (1998). In comparison, the best-fit Ω_m for flat models was found to be 0.31 ± 0.08 Choudhury et al. (2007). The flat concordance Λ CDM model remains an excellent fit to the Union2 data with the best-fit constant equation of state parameter $w = -0.997^{+0.050}_{-0.054}(\text{stat})^{+0.077}_{-0.082}(\text{stat+sys together})$ for a flat universe, or $w = -1.038^{+0.056}_{-0.059}(\text{stat})^{+0.093}_{-0.097}(\text{stat+sys together})$ with curvature Amanullah et al. (2010). Chaplygin gas is the more effective candidate of dark energy with equation of state $p = -B/\rho$ Kamenshchik et al. (2001) with $B > 0$. It has been generalized to the form $p = -B/\rho^\alpha$ Gorini et al. (2003) and thereafter modified to the form $p = A\rho - B/\rho^\alpha$ Debnath et al. (2004). The MCG best fits with the 3 year WMAP and the SDSS data with the choice of parameters $A = 0.085$ and $\alpha = 1.724$ Lu et al. (2008) which are improved constraints than the previous ones $-0.35 < A < 0.025$ Jun et al. (2005).

An effective explanation to the late cosmic acceleration can also be obtained by the modification of Einstein gravity. As a result various modified gravity theories came into existence. Brane-gravity is one such modified gravity theory that was established with the aim of modelling our present day universe in a better way, and consequently brane cosmology was developed. A review on brane-gravity and its various applications with special attention to cosmology is available in Rubakov et al. (2001); Maartens et al. (2004); Brax et al. (2004). Randall and Sundrum Randall1 et al. (1999); Randall2 et al. (1999) proposed a bulk-brane model to explain the higher dimensional theory, popularly known as RS II brane model. According to this model we live in a four dimensional world (called 3-brane, a domain wall)

which is embedded in a 5D space time (bulk). All matter fields are confined in the brane whereas gravity can only propagate in the bulk. The consistency of this brane model with the expanding universe has given immense popularity to this model of late.

Motivated by the previous works of some authors Ranjit et al. (2013); Chakraborty et al. (2012) here we assume the FRW universe in RS II model filled with the dark matter and the MCG type dark energy. Our basic idea is to determine the limits of the parameters involved in the EoS of MCG using the observational data. We present the Hubble parameter in terms of the observable parameters Ω_m , Ω_x and H_0 with the redshift z . From Stern data set (12 points), the bounds of the arbitrary parameters is obtained by minimizing the χ^2 test. The best-fit values of the parameters are obtained in 66%, 90% and 99% confidence levels. Via a joint analysis with BAO and CMB observations, we also obtain the bounds and the best fit values of the parameters (B, C) by fixing the other parameters A and α . From the best fit values of distance modulus $\mu(z)$ for our MCG model in RS II brane, we conclude that our model is in agreement with the union2 sample data.

The paper is organized as follows: In section 2, the basic equations and solutions for MCG in RS II braneworld is presented. The entire data analysis mechanism is given in section 3. Finally some observational conclusions are drawn in section 4.

2. Basic Equations and Solutions for MCG in RS II braneworld

In RS II model the effective equations of motion on the 3-brane embedded in 5D bulk having Z_2 -symmetry are given by Maartens et al. (2004); Maartens (2000); Randall et al. (1999); Shiromizu et al. (2000); Maeda et al. (2000); Sasaki et al. (2000)

$$^{(4)}G_{\mu\nu} = -\Lambda_4 q_{\mu\nu} + \kappa_4^2 \tau_{\mu\nu} + \kappa_5^4 \Pi_{\mu\nu} - E_{\mu\nu} \quad (1)$$

where

$$\kappa_4^2 = \frac{1}{6} \lambda \kappa_5^4, \quad (2)$$

$$\Lambda_4 = \frac{1}{2} \kappa_5^2 \left(\Lambda_5 + \frac{1}{6} \kappa_5^2 \lambda^2 \right) \quad (3)$$

and

$$\Pi_{\mu\nu} = -\frac{1}{4} \tau_{\mu\alpha} \tau_{\nu}^{\alpha} + \frac{1}{12} \tau \tau_{\mu\nu} + \frac{1}{8} q_{\mu\nu} \tau_{\alpha\beta} \tau^{\alpha\beta} - \frac{1}{24} q_{\mu\nu} \tau^2 \quad (4)$$

and $E_{\mu\nu}$ is the electric part of the 5D Weyl tensor. Here κ_5 , Λ_5 , $\tau_{\mu\nu}$ and Λ_4 are respectively the 5D gravitational coupling constant, 5D cosmological constant, the brane tension (vacuum energy), brane energy-momentum tensor and effective 4D cosmological constant. The explicit form of the above modified Einstein equations in flat universe are

$$3H^2 = \Lambda_4 + \kappa_4^2 \rho + \frac{\kappa_4^2}{2\lambda} \rho^2 + \frac{6}{\lambda \kappa_4^2} \mathcal{U} \quad (5)$$

and

$$2\dot{H} + 3H^2 = \Lambda_4 - \kappa_4^2 p - \frac{\kappa_4^2}{2\lambda} \rho p - \frac{\kappa_4^2}{2\lambda} \rho^2 - \frac{2}{\lambda \kappa_4^2} \mathcal{U} \quad (6)$$

The dark radiation \mathcal{U} obeys

$$\dot{\mathcal{U}} + 4H\mathcal{U} = 0 \quad (7)$$

Here $\rho = \rho_x + \rho_m$ and $p = p_x + p_m$, where ρ_m and p_m are the energy density and pressure of the dark matter with the equation of state given by $p_m = \omega_m \rho_m$ and ρ_x , p_x are respectively the energy density and pressure contribution of some dark energy. Here we consider an universe filled with Modified Chaplygin Gas (MCG). The equation of state(EOS) of MCG is given by

$$p_x = A\rho_x - \frac{B}{\rho_x^\alpha}, \quad B > 0, \quad 0 \leq \alpha \leq 1 \quad (8)$$

We also consider the dark matter and the dark energy are separately conserved and the conservation equations of dark matter and dark energy (MCG) are given by

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \quad (9)$$

and

$$\dot{\rho}_x + 3H(\rho_x + p_x) = 0 \quad (10)$$

From first conservation equation (9) we have the solution of ρ_m as

$$\rho_m = \rho_{m0}(1+z)^{3(1+\omega_m)} \quad (11)$$

From the conservation equation (10) we have the solution of the energy density as

$$\rho_x = \left[\frac{B}{A+1} + C(1+z)^{3(\alpha+1)(A+1)} \right]^{\frac{1}{\alpha+1}} \quad (12)$$

where C is the integrating constant, $z = \frac{1}{a} - 1$ is the cosmological redshift (choosing $a_0 = 1$) and the first constant term can be interpreted as the contribution of dark energy. So the above equation can be written as

$$\rho_x = \rho_{x0} \left[\frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{3(\alpha+1)(A+1)} \right]^{\frac{1}{\alpha+1}} \quad (13)$$

where ρ_{x0} is the present value of the dark energy density.

In the next section, we shall investigate some bounds of the parameters in RS II brane with the assumptions that $\Lambda_4 = \mathcal{U} = 0$ (i.e., in absence of cosmological constant and dark radiation) by observational data fitting. The parameters are determined by $H(z)$ - z (Stern), BAO and CMB data analysis Wu et al. (2007); Thakur et al. (2009); Paul et al. (2010, 2011); Ghose et al. (2011). We shall use the χ^2 minimization technique (statistical data analysis) to get the constraints of the parameters of MCG in RS II brane model.

3. Observational Data Analysis Mechanism

From the solution (13) of MCG and defining the dimensionless density parameters $\Omega_{m0} = \frac{\rho_{m0}}{3H_0^2}$ and $\Omega_{x0} = \frac{\rho_{x0}}{3H_0^2}$ we have the expression for Hubble parameter H in terms of redshift parameter z as follows ($8\pi G = c = 1$)

$$H(z) = H_0 \left[\kappa_4^2 \left\{ \Omega_{x0} \left(\frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{3(\alpha+1)(A+1)} \right)^{\frac{1}{\alpha+1}} + \Omega_{m0} (1+z)^{3(1+\omega_m)} \right\} \right. \\ \left. \left\{ 1 + \frac{3H_0^2}{2\lambda} \Omega_{x0} \left(\frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{3(\alpha+1)(A+1)} \right)^{\frac{1}{\alpha+1}} + \Omega_{m0} (1+z)^{3(1+\omega_m)} \right\}^{\frac{1}{2}} \right] \quad (14)$$

From equation (14), we see that the value of H depends on H_0, A, B, C, α, z so the above equation can be written as

$$H(z) = H_0 E(z) \quad (15)$$

where

$$E(z) = \left[\kappa_4^2 \left\{ \Omega_{x0} \left(\frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{3(\alpha+1)(A+1)} \right)^{\frac{1}{\alpha+1}} + \Omega_{m0} (1+z)^{3(1+\omega_m)} \right\} \right. \\ \left. \left\{ 1 + \frac{3H_0^2}{2\lambda} \Omega_{x0} \left(\frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{3(\alpha+1)(A+1)} \right)^{\frac{1}{\alpha+1}} + \Omega_{m0} (1+z)^{3(1+\omega_m)} \right\}^{\frac{1}{2}} \right]$$

$$\left\{ 1 + \frac{3H_0^2}{2\lambda} \Omega_{x0} \left(\frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{3(\alpha+1)(A+1)} \right)^{\frac{1}{\alpha+1}} + \Omega_{m0} (1+z)^{3(1+\omega_m)} \right\}^{\frac{1}{2}} \quad (16)$$

Now $E(z)$ contains four unknown parameters A, B, C and α . Now the relation between the two parameters will be obtained by fixing the other two parameters and by using observational data set. Eventually the bounds of the parameters will be obtained by using this observational data analysis mechanism.

z	$H(z)$	$\sigma(z)$
0	73	± 8
0.1	69	± 12
0.17	83	± 8
0.27	77	± 14
0.4	95	± 17.4
0.48	90	± 60
0.88	97	± 40.4
0.9	117	± 23
1.3	168	± 17.4
1.43	177	± 18.2
1.53	140	± 14
1.75	202	± 40.4

Table 1: The Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift z .

3.1. Analysis with Stern ($H(z)$ - z) Data Set

Using observed value of Hubble parameter at different redshifts (twelve data points) listed in observed Hubble data by Stern et al. (2010) we analyze the model. The Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift z are given in Table 1. For

this purpose we first form the χ^2 statistics as a sum of standard normal distribution as follows:

$$\chi_{Stern}^2 = \sum \frac{(H(z) - H_{obs}(z))^2}{\sigma^2(z)} \quad (17)$$

where $H(z)$ and $H_{obs}(z)$ are theoretical and observational values of Hubble parameter at different redshifts respectively and $\sigma(z)$ is the corresponding error for the particular observation given in table 1. Here, H_{obs} is a nuisance parameter and can be safely marginalized. We consider the present value of Hubble parameter $H_0 = 72 \pm 8 \text{ Kms}^{-1} \text{ Mpc}^{-1}$ and a fixed prior distribution. Here we shall determine the parameters A, B, C and α from minimizing the above distribution χ_{Stern}^2 . Fixing the two parameters C, α , the relation between the other parameters A, B can be determined by the observational data. The probability distribution function in terms of the parameters A, B, C and α can be written as

$$L = \int e^{-\frac{1}{2}\chi_{Stern}^2} P(H_0) dH_0 \quad (18)$$

where $P(H_0)$ is the prior distribution function for H_0 . We now plot the graph for different confidence levels. In early stage the Chaplygin Gas follow the equation of state $P = A\rho$ where $A \leq 1$. So, as per our theoretical model the two parameters should satisfy the two inequalities $A \leq 1$ and $B > 0$. Now our best fit analysis with Stern observational data support the theoretical range of the parameters. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in figures 1, 2 and 4 for $\alpha = 0.5$ and $A = 1, 1/3, -1/3$. The best fit values of B and C are tabulated in Table 2.

A	B	C	χ_{min}^2
1	0.5078000	0.114942	30.3789
$\frac{1}{3}$	0.0515799	0.764833	18.3400
$-\frac{1}{3}$	0.0226551	0.421103	8.5225

Table 2: $H(z)$ - z (Stern): The best fit values of B, C and the minimum values of χ^2 for different values of A .

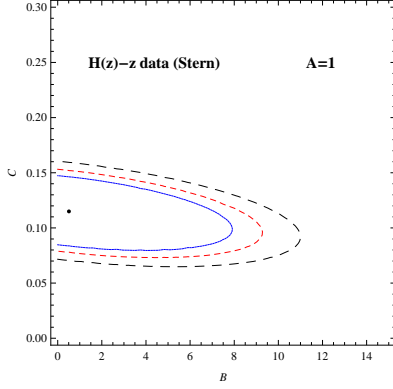


Fig.1

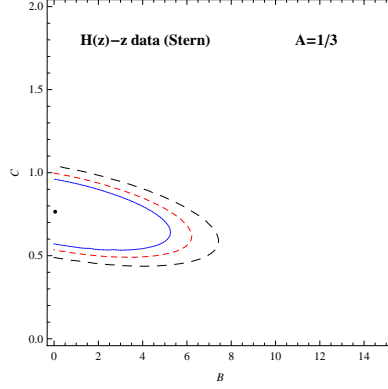


Fig.2

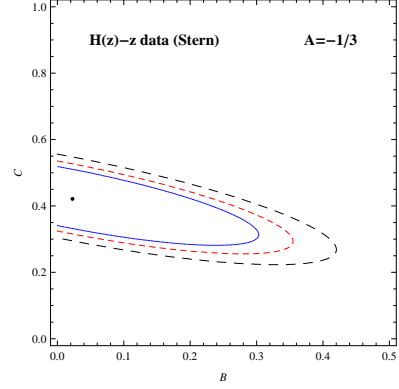


Fig.3

Fig.1 shows that the variation of C with B for $\alpha = 0.000001$, $\Omega_{m0} = 0.0013$, $\Omega_{x0} = 0.3688$ with $A = 1$ for different confidence levels. Fig.2 shows that the variation of C with B for $\alpha = 0.000001$, $\Omega_{m0} = 0.0012$, $\Omega_{x0} = 0.8035$ with $A = 1/3$ for different confidence levels. Fig.3 shows that the variation of C with B for $\alpha = 0.000001$, $\Omega_{m0} = 0.000064$, $\Omega_{x0} = 0.4551$ with $A = -1/3$ for different confidence levels. The contours are plotted for 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence level in these figures for the $H(z)$ - z (Stern) data analysis.

3.2. Joint Analysis with Stern + BAO Data Sets

The method of joint analysis, the Baryon Acoustic Oscillation (BAO) peak parameter value has been proposed by Eisenstein et al. (2005) and we shall use their approach. Sloan Digital Sky Survey (SDSS) survey is one of the first redshift survey by which the BAO signal has been directly detected at a scale ~ 100 Mpc. The said analysis is actually the combination of angular diameter distance and Hubble parameter at that redshift. This analysis is independent of the measurement of H_0 and not containing any particular dark energy. Here we examine the parameters B and C for Chaplygin gas model from the measurements of the BAO peak for low redshift (with range $0 < z < 0.35$) using standard χ^2 analysis. The error is corresponding to the standard deviation, where we consider Gaussian distribution. Low-redshift distance measurements is a lightly dependent on different cosmological parameters, the equation of state of dark energy and have the ability to measure the Hubble constant H_0 directly. The BAO peak parameter may be defined by

$$\mathcal{A} = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left(\frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right)^{2/3} \quad (19)$$

Here $E(z) = H(z)/H_0$ is the normalized Hubble parameter, the redshift $z_1 = 0.35$ is the typical redshift of the SDSS sample and the integration term is the dimensionless comoving distance to the redshift z_1 . The value of the parameter \mathcal{A} for the flat model of the universe is given by $\mathcal{A} = 0.469 \pm 0.017$ using SDSS data Eisenstein et al. (2005) from luminous red galaxies survey. Now the χ^2 function for the BAO measurement can be written as

$$\chi_{BAO}^2 = \frac{(\mathcal{A} - 0.469)^2}{(0.017)^2} \quad (20)$$

Now the total joint data analysis (Stern+BAO) for the χ^2 function may be defined by

$$\chi_{total}^2 = \chi_{Stern}^2 + \chi_{BAO}^2 \quad (21)$$

According to our analysis the joint scheme gives the best fit values of B and C in Table 3. Finally we draw the contours A vs B for the 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence limits depicted in figures 4–6 for $\alpha = 0.5$ and $A = 1, 1/3, -1/3$.

A	B	C	χ^2_{min}
1	0.960975	0.0242341	789.179
$\frac{1}{3}$	0.676543	0.0553636	775.823
$-\frac{1}{3}$	0.0221079	0.421197	769.474

Table 3: $H(z)$ - z (Stern) + BAO : The best fit values of B , C and the minimum values of χ^2 for different values of A .

3.3. Joint Analysis with Stern + BAO + CMB Data Sets

One interesting geometrical probe of dark energy can be determined by the angular scale of the first acoustic peak through angular scale of the sound horizon at the surface of last scattering which is encoded in the CMB power spectrum Cosmic Microwave Background (CMB) shift parameter is defined by Bond et al. (1997); Efstathiou et al. (1999); Nessaeris et al. (2007). It is not sensitive with respect to perturbations but are suitable to constrain model parameter. The CMB power spectrum first peak is the shift parameter which is given by

$$\mathcal{R} = \sqrt{\Omega_m} \int_0^{z_2} \frac{dz}{E(z)} \quad (22)$$

where z_2 is the value of redshift at the last scattering surface. From WMAP7 data of the work of Komatsu et al Komatsu et al. (2011) the value of the parameter has obtained as $\mathcal{R} = 1.726 \pm 0.018$ at the redshift $z = 1091.3$. Now the χ^2 function for the CMB measurement can be written as

$$\chi^2_{CMB} = \frac{(\mathcal{R} - 1.726)^2}{(0.018)^2} \quad (23)$$

Now when we consider three cosmological tests together, the total joint data analysis (Stern+BAO+CMB) for the χ^2 function may be defined by

$$\chi^2_{TOTAL} = \chi^2_{Stern} + \chi^2_{BAO} + \chi^2_{CMB} \quad (24)$$

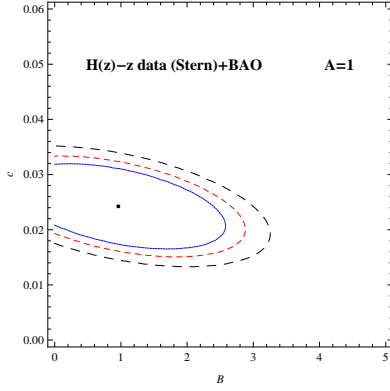


Fig.4

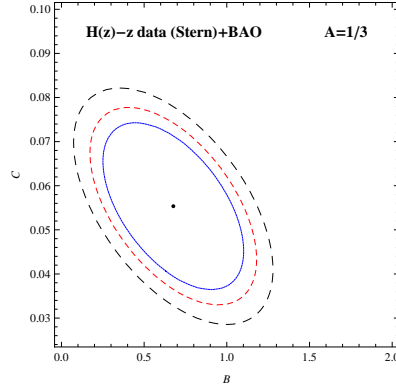


Fig.5

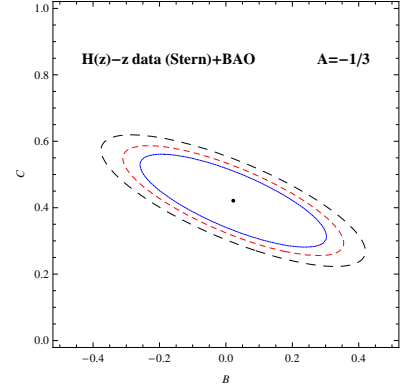


Fig.6

The contours are drawn for 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence levels for the $H(z)$ - z +BAO joint analysis. Fig.4 shows the variations of C against B for $\alpha = 0.000001, \Omega_{m0} = 0.01, \Omega_{x0} = 0.5091$ with $A = 1$. Fig.5 shows the variations of C against B for $\alpha = 0.000001, \Omega_{m0} = 0.01, \Omega_{x0} = 0.5627$ with $A = 1/3$. Fig.6 shows the variations of C against B for $\alpha = 0.000001, \Omega_{m0} = 0.01, \Omega_{x0} = 0.4544$ with $A = -1/3$.

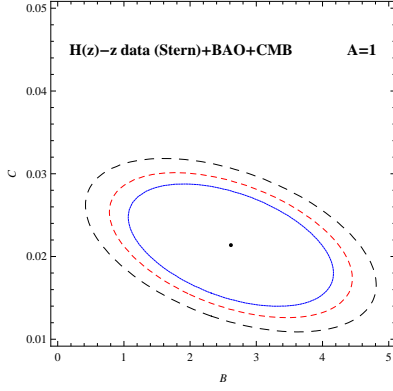


Fig.7

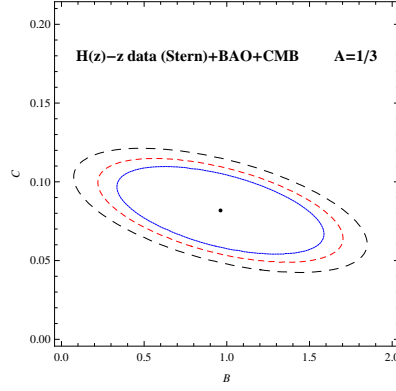


Fig.8

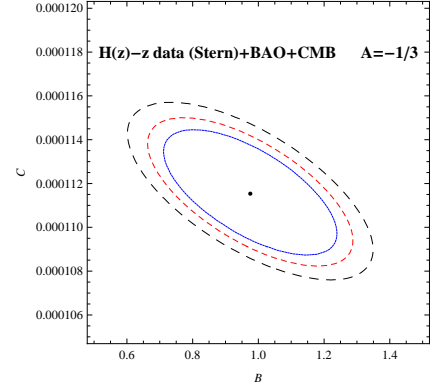


Fig.9

The contours are drawn for 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) confidence levels for the $H(z)$ - z +BAO+CMB joint analysis. Fig.7 shows the variations of C against B for $\alpha = 0.0001$, $\Omega_{m0} = 0.01$, $\Omega_{x0} = 1.3294$ with $A = 1$. Fig.8 shows the variations of C against B for $\alpha = 0.0001$, $\Omega_{m0} = 0.01$, $\Omega_{x0} = 0.8033$ with $A = 1/3$. Fig.9 shows the variations of C against B for $\alpha = 0.0001$, $\Omega_{m0} = 0.01$, $\Omega_{x0} = 1.4631$ with $A = -1/3$.

Now the best fit values of B and C for joint analysis of BAO and CMB with Stern observational data support the theoretical range of the parameters given in Table 4. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in figures 7-9 for $\alpha = 0.5$ and $A = 1, 1/3, -1/3$.

A	B	C	χ^2_{min}
1	2.616200	0.021372500	9979.820
$\frac{1}{3}$	0.961993	0.081817900	9970.610
$-\frac{1}{3}$	0.975391	0.000111536	882.179

Table 4: $H(z)$ - z (Stern) + BAO + CMB : The best fit values of B , C and the minimum values of χ^2 for different values of A .

3.4. Redshift-Magnitude Observations from Supernovae Type Ia

The Supernova Type Ia experiments provided the main evidence for the existence of dark energy. Since 1995, two teams of High- z Supernova Search and the Supernova Cosmology Project have discovered several type Ia supernovas at the high redshifts Perlmutter et al. (1998, 1999); Riess et al. (1998, 2004). The observations directly measure the distance modulus of a Supernovae and its redshift z Riess et al. (2007); Kowalaski et al. (2008). Now, take recent observational data, including SNe Ia which consists of 557 data points and belongs to the Union2 sample Amanullah et al. (2010).

From the observations, the luminosity distance $d_L(z)$ determines the dark energy density and is defined by

$$d_L(z) = (1+z)H_0 \int_0^z \frac{dz'}{H(z')} \quad (25)$$

and the distance modulus (distance between absolute and apparent luminosity of a distance object) for Supernovas is given by

$$\mu(z) = 5 \log_{10} \left[\frac{d_L(z)/H_0}{1 \text{ Mpc}} \right] + 25 \quad (26)$$

The best fit of distance modulus as a function $\mu(z)$ of redshift z for our theoretical model

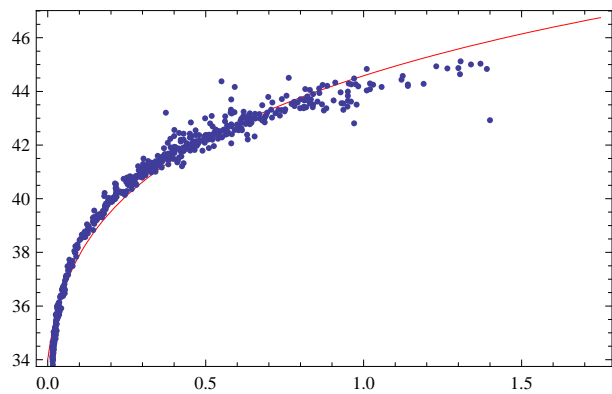


Fig.10

In fig.10, $u(z)$ vs z is plotted for our model (solid line) and the Union2 sample (dotted points).

and the Supernova Type Ia Union2 sample are drawn in figure 10 for our best fit values of α , A , B and C . From the curves, we see that the theoretical MCG model in LQC is in agreement with the union2 sample data.

4. Discussions

In this work, we have considered the FRW universe in RS II braneworld model filled with a combination of dark matter and dark energy in the form of modified Chaplygin gas (MCG). MCG is one of the candidate of unified dark matter-dark energy model. We present the Hubble parameter in terms of the observable parameters Ω_{m0} , Ω_{x0} and H_0 with the redshift z and the other parameters like A , B , C and α . We have chosen the observed values of $\kappa_4 = 0.1$, $\omega = -1.3$ and $H_0 = 72 \text{ Kms}^{-1} \text{ Mpc}^{-1}$. From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the χ^2 test. Next due to joint analysis of BAO and CMB observations, we have also obtained the best fit values and the bounds of the parameters (B, C) . We have plotted the statistical confidence contour of (B, C) for different confidence levels i.e., 66%(dotted, blue), 90%(dashed, red) and 99%(dashed, black) confidence levels by fixing observable parameters Ω_{m0} , Ω_{x0} and H_0 and some other parameters A and α for Stern, Stern+BAO and Stern+BAO+CMB data analysis.

From the Stern data, the best-fit values and bounds of the parameters (B, C) are obtained for $A(= 1, 1/3, -1/3)$, are shown in Table 2 and the figures 1-3 shows statistical confidence contour for 66%, 90% and 99% confidence levels. Next due to joint analysis with Stern + BAO data, we have also obtained the best-fit values and bounds of the parameters (B, C) for $A(= 1, 1/3, -1/3)$ and are shown in Table 3 and in figures 4-6 we have plotted the statistical confidence contour for 66%, 90% and 99% confidence levels. After that, due to joint analysis with Stern+BAO+CMB data, the best-fit values and bounds of the parameters (B, C) are found for $A(= 1, 1/3, -1/3)$, are shown in Table 4 and the figures 7-9 shows statistical confidence contour for 66%, 90% and 99% confidence levels. For each case, we compare the model parameters through the values of the parameters and by the statistical contours.

From this comparative study, one can understand the convergence of theoretical values of the parameters to the values of the parameters obtained from the observational data set and how it changes for different parametric values.

Finally the distance modulus $\mu(z)$ against redshift z has been drawn in figure 10 for our theoretical model of the MCG in RS II brane for the best fit values of the parameters and the observed SNe Ia Union2 data sample. So the observational data sets are perfectly consistent with our predicted theoretical MCG model in RS II brane.

The observational study discover the constraint of allowed composition of matter-energy by constraining the range of the values of the parameters for a physically viable MCG in RS II brane model. We have also verified that when λ is large, the best fit values of the parameters and other results of RS II brane model in MCG coincide with the results in Einstein's gravity Thakur et al. (2009). When λ is small, the best fit values of the parameters and the bounds of parameters spaces in different confidence levels in RS II brane distinguished from Einstein's gravity for MCG dark energy model. In summary, the conclusion of this discussion suggests that even though the quantum aspect of gravity have small effect on the observational constraint, but the cosmological observation can put upper bounds on the magnitude of the correction coming from quantum gravity that may be closer to the theoretical expectation than what one would expect.

Acknowledgments

The authors are thankful to IUCAA, Pune, India for warm hospitality where a part of the work was carried out.

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